

CFAR Analysis of the Multicoset-Thresholding Detector: Application to the Low Complexity Sub-Nyquist Radar Electronic Surveillance

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Abstract—Multicoset sampling scheme is a technique to achieve high-speed sampling rate, using a bank of lower-rate sampling channels. In this technique, each channel samples with a small delay with respect to the other channels. As a result, we often can reconstruct the high bandwidth input signal, by wisely combining the information from different channels. However, in many applications, the reconstruction is not the goal. Here, we consider an application, *i.e.* Radar Electronic Surveillance, in which the aim is the detection and identification of the incoming Radar pulses. As the sampling rate is very high, *e.g.* up to tens of Giga samples per second, we also need a fast detection scheme. We have recently proposed an efficient multicoset sampling technique, called LoCoMC, which is based on the thresholding for the detection and combining the information from different channels, to extract pulses. We here present an analytical investigation of the thresholding based detection and demonstrate how to choose the thresholding parameter. We then show that the algorithm can be competitive with a state of the art algorithm in performance, while it is computationally very cheap.

I. INTRODUCTION

There are many applications which need a very fast front-end analog to digital converters (ADC) before application of any digital signal processing. Some of such applications are spectrum sensing for cognitive radio [1] and electronic surveillance [2]. In these applications, we do not often use a full-band ADC for practical reasons, *e.g.* cost, weight and power consumption. The standard sub-optimal technique is based on the sweeping, where we each time only monitor some parts of the band and switch to another band in the next time period. Such an approach in Radar electronic surveillance is called Rapid Swept Superheterodyne Receiver (RSSR). However, sweeping techniques are not sensitive to the short-period events. As a result, many modifications have been proposed to compensate such a short-coming [2]. Most of such techniques are using multiple channels to simultaneously monitor a wider band. When the system has as many sub-ADC's as the downsampling factor, the ADC is called a time-interleaved ADC [3]. The implementation of a fast ADC needs accurate delay generation and a full-band Track and Hold (T&H) [4], [5]. Such an equivalence between the downsampling factor and the number of channels restricts us to a low downsampling ratio. On the other hand, we know that the signal of interest may have structures, which can be used to significantly reduce the number of necessary channels for successful signal reconstruction. The Multicoset

(MC) sampling scheme [6] is one of such approaches, which uses the sparsity of the signals in some transform domain, *e.g.* Fourier domain. Canonical multicoset sampling uses a subspace method to reconstruct the signal with a variant of the MUSIC technique [7]. An alternative technique is using an inspired method from compressed sensing, called spectrum blind MC sampling, when the signal is band-sparse [8], *i.e.* a few band out of whole spectrum is occupied. Both methods are computationally expensive, particularly considering desired sampling rates, *e.g.* multi-Giga samples per second. A new approach has been presented in [9], which breaks the large problem to some small size sparse approximation subproblems, and solves them with a fast greedy technique. In this approach, we still need to use an iterative method, which may not be easy to implement it in the real time.

We proposed a new low-computational complexity MC in [10], which is called a Low Complexity MultiCoset (LoCoMC) sampling. The front-end analog part of LoCoMC is similar to other multicoset samplers, which includes a delay and a sub-ADC in each channel and they sample with the same, but a fraction of the Nyquist rate, see [6] for more details on MC sampling framework. The selection of the delays here is different and follows a rule which provides us a nice property in the Harmonic frame which we use later for the sub-band classification. This part of the Multicoset sampler can be interpreted as a subselection of a time-interleaved ADC, for which some efficient fabrications have already been proposed, see for example [11]. As we only need fewer channels than a time-interleaved ADC, the hardware would be simpler, smaller and less power hungry.

The main difference between LoCoMC and other Multicoset sampling schemes is on the digital part, where we reconstruct the signal. As the aim is to reduce the computational complexity, we proposed a new fast reconstruction technique which is based on some simple linear transforms and low-complexity non-linear operators, *e.g.* absolute value and comparison. In an even more efficient implementation of the LoCoMC [12], we replaced the digital fractional delay and the time-frequency (TF) transform in the original framework [10], with a fractionally delayed TF transform which automatically accommodates the fractional delay and avoids the errors introduced by an independent DFD filter, see Figure 1. In the subband classifier, we received outputs of each channel, which are similar in magnitude and their phases are different.

Such phased differences help us to robustly identify the correct location of active area in the aliased signals and transfer it to the correct location in the original TF domain, *i.e.* de-alias the signal. To this end, a Harmonic frame has been used which can identify the exact locations, from noisy observations. We choose a set of delays such that the corresponding Harmonic frame is an Equiangular Tight Frame (ETF), to have a robust band allocation. The delay selection can be done off-line by exhaustive search as the size of the Frame is very small *i.e.* number of channels. It is worth mentioning that the recovery will be exact, if the signal has a property in the TF domain, called the Approximate Disjoint Aliased Support (ADAS) [10], where after the spectrum folding caused by aliasing, no two active TF bins overlaps. While this is not true for all signals, Radar ES signals are very sparse and as a result they satisfy this condition, with a high probability. For more detailed description of the LoCoMC framework, we refer the reader to [10] and [12].

We here discuss the back-end of an Radar early warning ES, where the task is to detect and identify incoming pulses. In such systems, it is not necessary to fully reconstruct the signal for the detection of pulses. This task is sometimes more difficult than a simple signal reconstruction. We will explain a new simple TF based detection and pulse centre identification, using a multicoset sampling scheme. A Constant False Alarm Rate (CFAR) analysis for the parameter selection is presented in this paper. A different CFAR analysis for the compressive measurement has been presented in [13], where a classical random Fourier subsampling was used as the sensing structure. As this approach needs to solve an ℓ_1 optimisation, the authors proposed to use complex approximate message passing (CAMP) algorithm. While CAMP is generally a fast solver, it is still an iterative technique, which would be difficult to have a real-time implementation in a multi-GHz sampling setting.

In the following, we first present the preliminary Mathematical tools for a constant false alarm analysis. We then explain how to detect TF-centres of the incoming pulses based on the Multicoset thresholding. We then explain how to calculate the threshold parameter. We finally compare the performance of the proposed technique with the Nyquist rate and RSSR type samplings.

A. Preliminaries

If the pdf's of random variables X_1 and X_2 are respectively $f_{X_1}(x)$ and $f_{X_2}(x)$, the pdf of their sum $\hat{X} := X_1 + X_2$ can be found by [14],

$$\begin{aligned} f_{\hat{X}}(\hat{x}) &= \int_{-\infty}^{+\infty} \int_{-\infty}^{\hat{x}-x_2} f_{X_1}(x_1) f_{X_2}(x_2) dx_1 dx_2 \\ &= \int_{-\infty}^{+\infty} f_{X_1}(\hat{x} - x_2) f_{X_2}(x_2) dx_2 \\ &= (f_{X_1} * f_{X_2})(\hat{x}) \end{aligned}$$

The convolution can be more efficiently calculated in the inverse Fourier domain as follows,

$$F_{\hat{X}}(\hat{\omega}) = F_{X_1}(\omega) F_{X_2}(\omega) \quad (1)$$

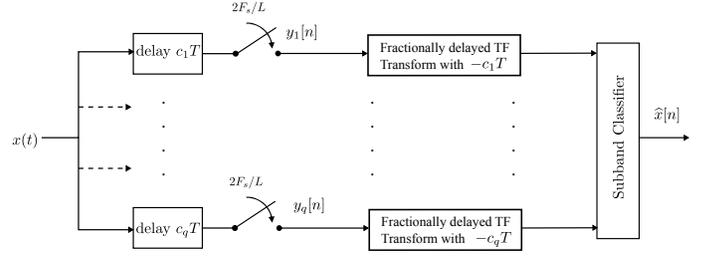


Fig. 1. Low-Complexity sub-Nyquist sampling system (LoCoMC).

where F_{X_1} and F_{X_2} are respectively the characteristic functions of f_{X_1} and f_{X_2} , and then calculate Fourier transform of $F_{\hat{X}}(\hat{\omega})$.

II. MULTICOSET THRESHOLDING DETECTION

In a Multicoset sampling system, we have delayed versions of the input signal. Such a delay appears as a change in the phase of the aliased signals. On the other hand, sub-ADC's sample at different time instances. We can then assume that the additive noise in different channels are independent, *e.g.* *i.i.d.* zero-mean Gaussian. Similar to the Nyquist rate TF detection, we calculate the STFT's of Multicoset signals. We recall that applying such STFT's would be much simpler than applying it to the Nyquist rate sampled signals. The reason is in the dimensionality reduction of the downsampling operation, which generates aliasing signals.

When the signal is sampled with the Nyquist rate, we can simply apply a threshold to the STFT coefficient magnitudes and identify the active areas in the TF plain. While we can similarly apply this process to a single channel, it will practically be better to use the information from all channels, to average out the noise. For this reason, we need to calibrate Multicoset signals, by inverse delaying each channel, with the corresponding delay factor. Such a process is automatically embedded in LoCoMC, when we use Fractionally delayed TF transforms.

In the next stage of the detection/identification, we need to combine connected/closed active areas, to identify the total active area of each pulse. We also need to cancel isolated active cells in the TF plain and remove some noise artefacts. We also note the centre of each group of connected/closed active bins, as the centre of a pulse. In this case, we can have a comparison between different approaches in the TF pulse centres recovery.

To have a reliable pulse detection, we need the energy of each pulse be generally larger than the noise level. Based on using a local noise level estimation or some assumptions about the noise level, we can theoretically find a thresholding level that the noise energy is often lower than individual pulse energy, but with the given rate of false alarms. In the next section, we use the standard CFAR analysis for the ES [2], and extend it for the Multicoset-thresholding based detection/identification.

III. CONSTANT FALSE ALARM RATE ANALYSIS

The analysis of false alarm for the Gaussian I and Q input channels in the Radar ES [2], will be derived in some steps. We separately investigate the effect of each part of the Multicoset-thresholding detector, and combine them all

to derive a threshold, which guarantees a given CFAR. We assume that the additive noise on each sample has Gaussian real and imaginary parts. If X and Y are *i.i.d.* Gaussian random processes, with the probability density functions (pdf),

$$f_X(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{x^2}{2\sigma^2}}, \quad f_Y(y) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{y^2}{2\sigma^2}},$$

we can formulate an instance of the measurement noise process Z as follows,

$$\begin{aligned} z &= x + iy \\ &= re^{-i\phi} \end{aligned}$$

where $0 \leq r$ and $0 \leq \phi \leq 2\pi$ are respectively the magnitude and phase of z and $i = \sqrt{-1}$. r and ϕ can be respectively interpreted as instances of random processes R and Φ . It is easy to check that R has a Rayleigh distribution in this setting, *e.g.* see [2, Chapter 9].

A. Multichannel and Complex Combination of Noises:

If we have q channels in the Multicoset sampling system, we simply add the corresponding values of channels to average over the noise. In this setting, we can derive the pdf of the real part of noise, using the formulation of (1) as follows,

$$f_X(x) = f_X\left(\sum_{l=1}^q x_l\right) = \frac{1}{\sqrt{2q\pi}\sigma} e^{-\frac{x^2}{2q\sigma^2}}, \quad (2)$$

where $x = \sum_{l=1}^q x_l$ is the total noise of one channel. The pdf of the imaginary part can be calculated similar to (2). We can now calculate $f_R(r)$ by marginalising $f_X(x)f_Y(y)$ over ϕ as follows,

$$\begin{aligned} f_R(r) &= \int_0^{2\pi} r f_X(r \cos \phi) f_Y(r \sin \phi) d\phi \\ &= \int_0^{2\pi} \frac{r}{2\pi q\sigma^2} e^{-\frac{(r^2 \cos^2 \phi + r^2 \sin^2 \phi)}{2q\sigma^2}} d\phi \\ &= \int_0^{2\pi} \frac{r}{2\pi q\sigma^2} e^{-\frac{r^2}{2q\sigma^2}} d\phi \\ &= \frac{r}{q\sigma^2} e^{-\frac{r^2}{2q\sigma^2}}. \end{aligned}$$

B. Windowed Fourier Signals:

We now investigate the effect of windowing and transforming into the Fourier domain. Let the window be presented by a vector $\mathbf{w} = [w_l]_{1 \leq l \leq N}$, for a length- N window. As the window is applied first, *i.e.* $z_i = w_i x_i$, we can derive the effect of windowing on $X \sim \mathcal{N}(0, \sigma_x)$, as follows,

$$\begin{aligned} \mathbb{E}\{z_i^H z_i\} &= w_i^2 \mathbb{E}\{x^H x\} \\ &= w_i^2 \sigma_x^2 = (w_i \sigma_x)^2 \end{aligned}$$

The k th element of the Fourier transform of the windowed signal z can be calculated as follows,

$$\bar{z}_k = \sum_{j=1}^N e^{-\frac{i2\pi k j}{N}} z_j$$

where bar sign indicates that the signal is in the Fourier domain. We can then calculate the power spectral density as follows,

$$\begin{aligned} \mathbb{E}\{\bar{z}_k^2\} &= \sum_{j'=1}^N \sum_{j=1}^N e^{-\frac{-i2\pi k(j-j')}{N}} \mathbb{E}\{z_j^H z_{j'}\} \\ &= \sum_{j'=1}^N \sum_{j=1}^N e^{-\frac{-i2\pi k(j-j')}{N}} w_j^* w_{j'} \mathbb{E}\{x_j^H x_{j'}\} \\ &= \sigma^2 \sum_{j=1}^N w_j^2 \end{aligned} \quad (3)$$

To derive the third equation in (3) we used the fact that x_j and $x_{j'}$ are independent for $j \neq j'$. To have an analogy with the case that there is a rectangular window, we normalise the window to have $\sum_{j=1}^N w_j^2 = N$. We can then have $\mathbb{E}\{\bar{z}_k^2\} = N\sigma_x^2$. In other words, using a rectangular window magnifies the variance with a factor of $\mathcal{O}(N)$, but, of course the amplitude of a single tone can be magnified by N , leading to the well-known result that the signal to noise ratio in this case is improved by a factor of N . Other windows will spread the signal across multiple elements of the Fourier transform, reducing the gain slightly as tabulated in [15].

C. Threshold Setting

In the multicoset sampling, we have a bank of downsampled versions of the signals. If we downsample with a factor of L , the window length would be reduced to N/L , to observe the same structure in the Frequency domain. As it was mentioned earlier, we add the signals from different channels before applying the Frequency domain thresholding detection.

Let we have q channels, each downsampled with a factor of L . We can use the standard false alarm analysis to derive the probability of false alarms P_{fa} as follows,

$$\begin{aligned} P_{fa} &= \int_{\tau}^{\infty} f_R(r) \Big|_{\sqrt{N}\sigma} dr \\ &= \int_{\tau}^{\infty} \frac{rL}{qN\sigma^2} e^{-\frac{Lr^2}{2qN\sigma^2}} dr \\ &= e^{-\frac{L\tau^2}{2qN\sigma^2}} \end{aligned}$$

where τ is the thresholding parameter. We can then easily calculate τ for a fixed probability of false alarm as follows,

$$\tau = \sqrt{-2 \left(\frac{q}{L}\right) N\sigma^2 \ln P_{fa}}. \quad (4)$$

IV. SIMULATIONS

The setting for evaluating the proposed Multicoset-thresholding detector, is very similar to [10]. We used the pulse information provided by Thales UK, which is a set of pulse descriptor words in X-band. For simplification, we demodulated the active bandwidth of 1.4 GHz to the baseband and used $q = 4$ sub-ADC's. We also downsampled with a factor of $L = 13$, *i.e.* $1400/13 \approx 108$ MHz, which approximately provides the total undersampling rate of 3. STFT used

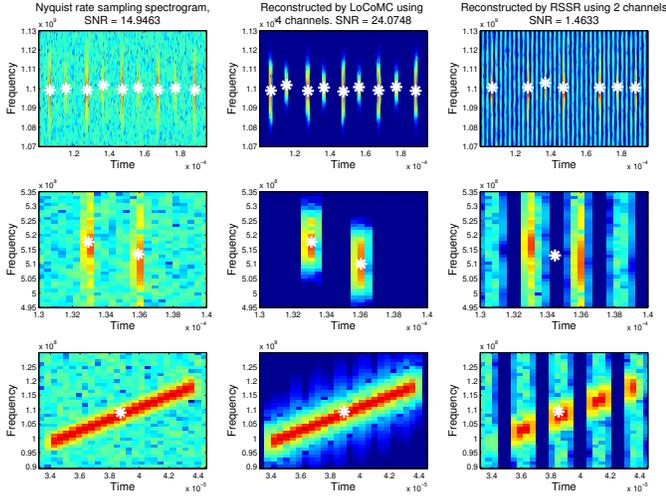


Fig. 2. Time-Frequency plots of recovered signals and pulses TF centres (indicated by stars). Nyquist rate system (left column), LoCoMC (middle column) and RSSR (right column)

a Hann (cosine-squared) window [15] of length $N = 1014$, *i.e.* 13×78 , with half length overlapping between consecutive windows. Each STFT therefore contains data from a period of 724 ns. The set of delays in the Multicoset system was selected $c = [6 \ 7 \ 10 \ 12]$, which generates a Harmonic equiangular tight frame of size 4×13 . The input SNR was 14.94 dB. We also selected the ratio between the magnitudes of the largest to the smallest pulses to be twenty.

The proposed Multicoset sampling scheme generates 78 opportunities for a false alarm every STFT. With 50% overlap, this corresponds to about 2×10^8 opportunities for a false alarm per second so a false alarm probability of $P_{fa} = 10^{-6}$ would be expected to give rise to about 200 false pulses per second.

For comparison, we used an RSSR system, which used 2 sub-ADC's, with overlapping in time to compensate the windowing artefacts, each with 6-times undersampling. This gives an overall undersampling rate of 3. We assumed an ideal switch-over between different bands in RSSR, *i.e.* no delay, to eliminate the switch-over error. We also used a full Nyquist rate sampling system as a reference. We have shown three different parts of the signal in Figure 2, which are a train of pulses (top row), two closed individual pulses (middle row) and a linear chirp (bottom row). The outputs of the Nyquist, LoCoMC and RSSR detectors are respectively shown from left to right. The identified pulse centres are shown with white stars. In this experiment, the reconstructed signal using LoCoMC has a higher SNR, which is due to noise cancellation, and all pulses are correctly recovered. In a contrast, RSSR fails to recover some of pulses and reduces the processing gain in long pulses and chirps. It is also clear that we need to combine sets of active TF bins with larger distances to compensate the TF sweeping artefact, *i.e.* missing some parts of the long pulses/chirps by looking to another part of TF domain. While this helps us to successfully recover the centre of a chirp, *i.e.* bottom right, we misidentify the pulses in the middle-right panel, where two pulses are closed.

We ran the simulations with a slightly longer signal, with the input SNR of 20.97 dB. If we use the thresholding

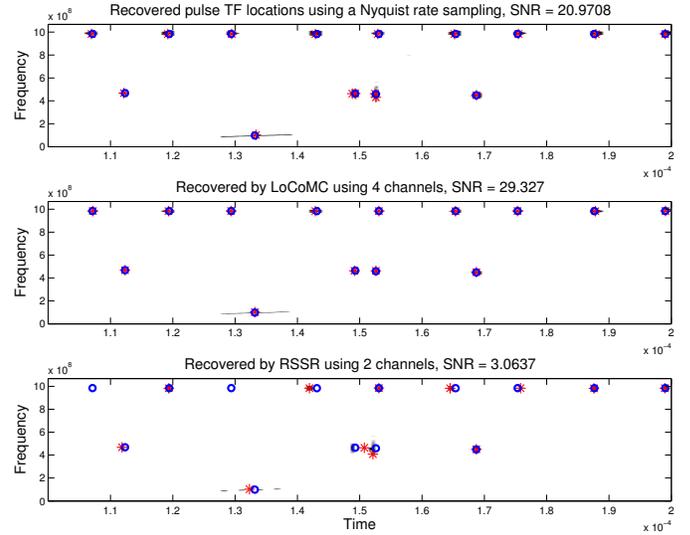


Fig. 3. Time-Frequency locations of pulses: original location (stars) and identified locations (circles). Nyquist rate, LoCoMC and RSSR methods are respectively shown on the top, middle and bottom panels.

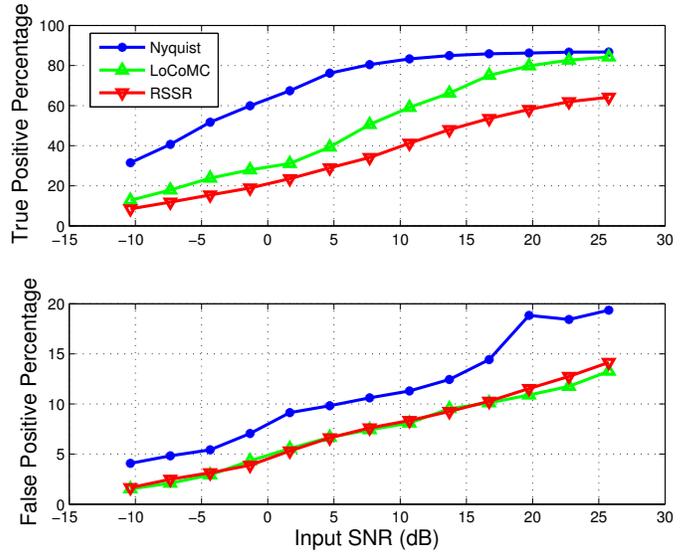


Fig. 4. Average True Positive (top) and False Positive (bottom) figures with different input SNR's.

detection with the explained techniques, *i.e.* Nyquist, LoCoMC and RSSR, we get the results showing in Figure 3, where the original TF location of the pulses/chirps are shown with stars and detected locations are shown with circles. This figure shows that the proposed Multicoset detection algorithm, with the derived threshold in (4), can precisely recover the exact locations in high SNR.

Sometimes we are interested to use the Radar ES early warning systems in the low-SNR scenario. We already know that the noise-folding artefact reduces the effective SNR in Multicoset and compressive sensing systems [16]. We therefore repeated the simulations for a very long signal, *i.e.* more than fifty thousand pulses (about 350 ms), to farther investigate such a setting. As there are many circumstances that the

recovery of TF pulse centres are challenging, all sampling techniques fail to correctly detect some pulses. This cases include overlapping pulses, too close pulses and short pulses in very noisy measurements. In a very low SNR, some of the pulses will be buried in the noise, particularly in the Multicoset and CS based systems. We have respectively plotted the actual true positive and false positive rates in the top and bottom panels of Figure 4. The performance of LoCoMC locates between the two compared techniques, while its complexity is roughly the same as RSSR. We observe that, while for high SNR the performance is close to the Nyquist rate sampling setting, it is getting closed to the RSSR performance for low SNR's.

The percentage of false positives is the number of false positives expressed as a percentage of the number of pulses actually present in the data set. Note that we would expect about 70 false positive due to noise over 350ms, i.e. a false positive rate due to noise of 0.14%. In fact most of the false positives are due to spurious detections associated with the edges (in either time or frequency) of genuine pulses. This explanation is supported by the fact that the false pulse rate increases with signal to noise ratio, whereas the rate of false pulses due to noise will be independent of the signal to noise ratio.

V. CONCLUSION

A new method was proposed here to reduce the complexity of high speed Radar ES early warning systems, using the signal models and a Multicoset sampling scheme. We here presented a thresholding based detection algorithm and derive the relation between threshold parameter and CFAR. We can then use the optimal parameter for the Radar pulse detection and identification. We presented the statistical figures of the proposed algorithm and compared with Nyquist rate system and a well-known technique, *i.e.* RSSR. The performance of the LoCoMC is comparable with the Nyquist rate system in a high SNR setting, while it is computationally simpler. LoCoMC also performs better than RSSR in all investigated simulations, which may be due to the continuously monitoring property of LoCoMC.

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